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THE CALCULATION OF DINAMIC DURABILITY OF ELASTIC DEFORMED SISTEMS

The deformed systems with distributed parameters are often used in the equipments of different branch of industry. There are rod structures with distributed masses witch have loads from interaction with other objects as well as localized masses that determined inertial loads. The examples of such systems with distributed / localized mass are shafts of rolling mills, transport pipelines of overland equipment and deep-water mining complexes, airlifting and pumping systems, boring flight of drilling rigs for oil and gas wells and of pit-shafts and special-purpose wells of big diameters, pipelines of drain and ventilating systems, pipelines of suction-tube dredge, and others.

Dynamic processes of such systems are described by differential equation in partial derivative, the solutions are presented as eigenfunctions. The eigenfunctions are always orthogonal at the absence localized masses, but the eigenfunctions are weighted orthogonal for the systems with step-variable section. This fact essentially complicates the solution of dynamic tasks.

Transverse vibrations of homogeneous rods under difference boundary conditions are specified in the monograph [1]. Similar problems about natural vibrations are discussed in [2] for double-step rod. Various tasks about dynamics of homogeneous rods with localized masses are described in [3].

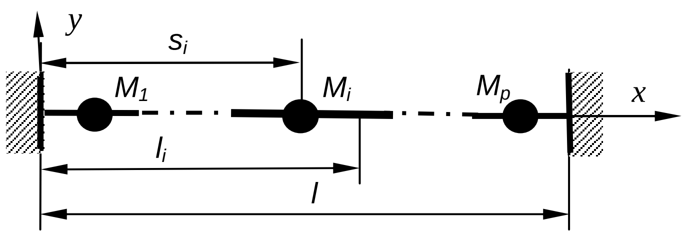

Fig.1. Calculation scheme of rod system

Let's examine the general task about transverse vibrations of the rod systems of step-variable hardness with localized masses $M_i (i=1,2,...,p)$ (fig. 1) [4].

Transverse vibrations of such system section is examined separately for each parts and then conjugation conditions are used for the system parts

$$\frac{\partial^2 y_i}{\partial t^2} + a_i^2 \frac{\partial^4 y_i}{\partial x^4} = 0, \quad i = 1, 2,..., p, \qquad (1)$$

where $y_i(x,t)$ - transverse displacement of rod sections *i*- part, $a_i^2 = \frac{E_i J_i}{m_i}$, $E_i J_i$, m_i – bending harshness and linear mass of the parts correspondently.

At first we study natural vibrations of such system without localized mass. Boundary conditions should be specified for the solution of the equation (1)

$$L_1 y_1\Big|_{x=0} = 0;\ L_2 y_1\Big|_{x=0} = 0;\ L_3 y_p\Big|_{x=l} = 0;\ L_4 y_p\Big|_{x=l} = 0. \qquad (2)$$

The type of liner differential abstract function $L_j (j = 1,...,4)$ at the condition (2) corresponds to following way of rod system ends fixing: rigid fixing, pinning and free end. Moreover, it is necessary to specify the connection conditions at $x = l_i (i = 1, 2, 3..., p-1)$.

$$\begin{aligned} y_i(l_i,t) &= y_{i+1}(l_i,t); \\ y_i'(l_i,t) &= y_{i+1}'(l_i,t); \\ E_i J_i y_i''(l_i,t) &= E_{i+1} J_{i+1} y_{i+1}''(l_i,t); \\ E_i J_i y_i'''(l_i,t) &= E_{i+1} J_{i+1} y_{i+1}'''(l_i,t). \end{aligned} \qquad (3)$$

Let's study the properties of the eigenfunctions, because it is necessarily for the solution of tasks on natural and forced vibrations. The eigenfunctions of the boundary problem are specified as

$$X_n(x) = \sum_{i=1}^{p} (e(l_i - x) - e(l_{i-1} - x)) X_{n,i}(x), \qquad (4)$$

where $e(x)$ - unit function, $X_{n,i}(x)$ - the eigenfunctions of corresponding boundary problem (1).

Make use of well known formula [1] for each part

$$\left(\omega_n^2 - \omega_m^2\right) \int_{l_{i-1}}^{l_i} X_{n,i} X_{m,i} dx = a_i^2 \left(X_{m,i} X_{n,i}''' - X_{n,i} X_{m,i}''' + X_{n,i}' X_{m,i}'' - X_{m,i}' X_{n,i}''\right)\Big|_{l_{i-1}}^{l_i}, \qquad (5)$$

where ω_n - natural frequency of vibration.

The integrals (5) are summed over whole system. This sum (5) are equal to zero for extreme parts of the system because of the boundary condition (2). Using the connection condition (3) we obtain

$$\begin{aligned} \left(\omega_n^2 - \omega_m^2\right) \int_0^l X_n X_m dx = \sum_{i=1}^{p-1} E_i J_i \left(\frac{1}{m_i} - \frac{1}{m_{i+1}}\right) (X_{m,i}(l_i) X_{n,i}'''(l_i) - X_{n,i}(l_i) X_{m,i}'''(l_i) + \\ + X_{n,i}'(l_i) X_{m,i}''(l_i) - X_{m,i}'(l_i) X_{n,i}''(l_i)). \end{aligned} \qquad (6)$$

It is evident from (6) that the orthogonality of the eigenfunctions is possible only if linear masses is equal $m_i = m_{i+1}$; else they are orthogonal with weight

$$\rho_1(x) = \sum_{i=1}^{p} m_i (e(l_i - x) - e(l_{i-1} - x)). \qquad (7)$$

When let's study the influence of local masses on the ortogonality of the eigenfunctions of concerned boundary problem. Presence of local mass M_i at $x = s_i$ leads to changing last equation of (3) to the following

$$E_i J_i \left(y'''(s_i + 0, t) - y'''(s_i - 0, t) \right) = M_i \ddot{y}(s_i, t). \qquad (8)$$

The condition (8) is put though the eigenfunctions

$$X'''_{n,i}(s_i + 0) - X'''_{n,i}(s_i - 0) = -\frac{\omega_n^2 M_i}{E_i J_i} X_{n,i}(s_i). \qquad (9)$$

If integral (5) is put as sum of two integral over interval $[l_{i-1}; s_i - 0]$ and $[s_i + 0; l_i]$, we obtain the following equation from formulas (5) and (9) at the condition $\omega_n \neq \omega_m$

$$\int_{l_{i-1}}^{l_i} X_{n,i} X_{m,i} dx = -\frac{M_i a_i^2}{E_i J_i} X_{m,i}(s_i) X_{n,i}(s_i). \qquad (10)$$

The equation (10) signifies the ortogonality of the eigenfunctions with weight in this case on the segment $[l_{i-1}; l_i]$

$$\rho_{2,i}(x) = m_i + M_i \delta(x - s_i), \qquad (11)$$

where $\delta(x)$ - Dirak delta function.

The ortogonality with weight of the eigenfunctions of given boundary problem is obtained combining the results (7) and (11)

$$\rho(x) = \sum_{i=1}^{p} (m_i + M_i \delta(x - s_i))(e(l_i - x) - e(l_{i-1} - x)). \qquad (12)$$

The formula (12) for weight corresponds to general theory of the eigenfunctions [5].

The squared norm of the eigenfunctions with weight is defined like

$$\int_0^l \rho(x) X_n^2(x) dx = \sum_{i=1}^{p} m_i \int_{l_{i-1}}^{l_i} X_{n,i}^2(x) dx + M_i X_{n,i}^2(s_i). \qquad (13)$$

It is necessarily to introduce wave numbers $k_{n,i}^4 = \omega_n^2 / a_i^2$ and turn to differentiation over $z = k_{n,i} x$ for using well-known equation for calculation the squared norm of the eigenfunctions [1]. Then the connection conditions (3) for the eigenfunctions assume the form

$$
\begin{aligned}
&X_{n,i}(l_i) = X_{n,i+1}(l_i);\\
&k_{n,i}X'_{n,i}(l_i) = k_{n,i+1}X'_{n,i+1}(l_i);\\
&k_{n,i}^2 E_i J_i X''_{n,i}(l_i) = k_{n,i+1}^2 E_{i+1} J_{i+1} X''_{n,i+1}(l_i);,\\
&k_{n,i}^3 E_i J_i X'''_{n,i}(l_i) = k_{n,i+1}^3 E_{i+1} J_{i+1} X'''_{n,i+1}(l_i),
\end{aligned}
\qquad (14)
$$

where the differentiation is done over variable z.

The case of rigid fixing of the rod system ends is examined below as an example. We obtain the equation for the squared norm (14) taking into consideration the boundary conditions

$$
\begin{aligned}
\int_0^l \rho(x)X_n^2(x)dx = &\sum_{i=1}^{p-1}\frac{3}{4}X_{n,i}(l_i)\left(\frac{m_i}{k_{n,i}}X'''_{n,i}(l_i) - \frac{m_{i+1}}{k_{n,i+1}}X'''_{n,i+1}(l_i)\right) - \frac{l_i}{2}(m_i X'_{n,i}(l_i)X'''_{n,i}(l_i) -\\
&- m_{i+1}X'_{n,i+1}(l_i)X'''_{n,i+1}(l_i)) + \frac{l_i}{4}(m_i X_{n,i}^2(l_i) - m_{i+1}X_{n,i+1}^2(l_i)) -\\
&- \frac{1}{4}\left(\frac{m_i}{k_{n,i}}X'_{n,i}(l_i)X''_{n,i}(l_i) - \frac{m_{i+1}}{k_{n,i+1}}X'_{n,i+1}(l_i)X''_{n,i+1}(l_i)\right) +\\
&+ \frac{l_i}{4}\left(m_i\left(X''_{n,i}(l_i)\right)^2 - m_{i+1}\left(X''_{n,i+1}(l_i)\right)^2\right) + \frac{lm_p}{4}\left(X''_{n,p}(l_i)\right)^2 + \sum_{i=1}^{p} M_i X_{n,i}^2(s_i).
\end{aligned}
\qquad (15)
$$

The equation (15) assumes the following form taking into account connection conditions (14) and turning to differentiation over x

$$
\begin{aligned}
\int_0^l \rho(x)X_n^2(x)dx = &\sum_{i=1}^{p-1}\frac{E_i J_i l_i}{4\omega_n^2}\left(1 - \frac{E_i J_i}{E_{i+1}J_{i+1}}\right)\left(\left(X''_{n,i}(l_i)\right)^2 - 2X'_{n,i}(l_i)X'''_{n,i}(l_i)\right) +\\
&+ \frac{m_i l_i}{4}X_{n,i}^2(l_i)\left(1 - \frac{m_{i+1}}{m_i}\right) + \frac{E_p J_p l}{4\omega_n^2}\left(X''_{n,p}(l)\right)^2 + \sum_{i=1}^{p} M_i X_{n,i}^2(s_i).
\end{aligned}
\qquad (16)
$$

The squared norm of the eigenfunctions is defined similarly for other way of system end fixing. For example penultimate member in (16) should be replaced by

$-\frac{E_p J_p l}{2\omega_n^2}X'_{n,p}(l)X'''_{n,p}(l)$ in the case of pinning and by $\frac{1}{4}m_p l X_{n,p}^2(l)$ for the free end.

It is easy to obtain from (16) well-known equation for the squared norm of the eigenfunctions with weight $\rho(x) = 1$ [1] in the case of homogenous rod.

In much the same way it is possible to examine corresponding tasks for longitudinal vibrations of step-variable section rod system with localized masses. In this case the equation of longitudinal vibrations of i- part is given by

$$\frac{\partial^2 u_i}{\partial t^2} = a_i^2 \frac{\partial^2 u_i}{\partial x^2},$$

where $u(x,t)$ - longitudinal displacement, $a_i^2 = E_i F_i / m_i$, $E_i F_i, m_i$ - correspondently longitudinal harshness and linear mass of the system.

The equation for the eigenfunctions is obtained from conjugation conditions at $x = l_i$

$$\begin{aligned} &X_{n,i}(l_i) = X_{n,i+1}(l_i); \\ &E_i F_i X'_{n,i}(l_i) = E_{i+1} F_{i+1} X'_{n,i+1}(l_i). \end{aligned} \tag{17}$$

The characteristic of the eigenfunctions is the following in concerned case (Fig. 2)

$$\left(\omega_n^2 - \omega_m^2\right)\int_0^l X_n X_m dx = \sum_{i=1}^{p-1} E_i F_i \left(\frac{1}{m_i} - \frac{1}{m_{i+1}}\right)(X_{n,i}(l_i) X'_{m,i}(l_i) - X_{m,i}(l_i) X'_{n,i}(l_i)). \tag{18}$$

It results from equation (18) and the conjugation conditions (17) that the eigenfunctions are also orthogonal with weight (12) taking into account localized masses.

The squared norm of the eigenfunctions is calculated as

$$\begin{aligned} \int_0^l \rho(x) X_n^2(x) dx = \frac{1}{2} \sum_{i=1}^{p-1} \frac{E_i F_i}{\omega_n^2} X'_{n,i}(l_i) \left(l_i X'_{n,i}(l_i) \left(1 - \frac{m_i}{m_{i+1}}\right) - X_{n,i}(l_i) \left(1 - \frac{a_{i+1}}{a_i}\right)\right) + \\ + m_i l_i X_{n,i}^2(l_i) \left(1 - \frac{m_{i+1}}{m_i}\right) + \frac{E_p F_p l}{2\omega_n^2} \left(X'_{n,p}(l)\right)^2 + \sum_{i=1}^{p} M_i X_{n,i}^2(s_i). \end{aligned} \tag{19}$$

The equation (19) corresponds to fixing end of the rod system.

Penultimate member in (19) should be replaced by $\frac{l}{2} X_{n,p}^2(l)$ in the case of free end.

Taking into account that the mathematical model for torsional vibrations totally coincides with the model for longitudinal vibrations the equations (18) and (19) are correct with substitution of hardness at tension (compression) on corresponding hardness at torsion.

It is necessary to define the eigenfunctions, use the conjugation conditions and the boundary conditions for examination of natural vibrations. Homogeneous system of linear algebraic equation with $4p$ $(2p)$ unknowns is obtained as a result. The equation for determination of natural vibration frequency is defined if the determinate of indicated system is equated to zero [2].

The Fourier method for the eigenfunctions with weight (12) could be used for examination of forced vibrations and application of the formula (19) simplifies this problem.

Obtained results could be used for rough calculation method of dynamic of variable section rod system if the form of this rod section is approximated as step figure.

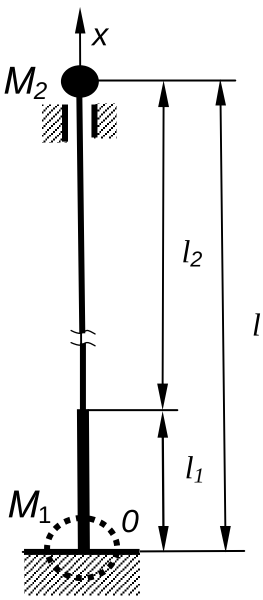

Fig. 2. Calculation scheme of drill column

The task about longitudinal impact during double-step drill column descending is examined in detail as an example (fig/ 2)[6].

The boundary and initial conditions (2) take the following forms in this case

$$M_2\ddot{u}(l,t)+E_2F_2u'(l,t)=0; \quad u(0,t)=0. \qquad (20)$$

$$u(x,0)=0; \quad \dot{u}(x,0)=-V_o. \qquad (21)$$

Here M_1- mass of cutter with weight; M_2 - tackle system mass; v_o - descending velocity of column in coalface: $k_{ni}=\omega_n/a_i$.

The weight function and the eigenfunctions have the forms:

$$\rho(x)=\begin{cases} m_1, & 0\le x\le l_1, \\ m_2+M_2\delta(x-l), & l_1<x\le l; \end{cases}$$

$$X_{n,i}(x)=A_{n,i}\cos k_{n,i}x+B_{n,i}\sin k_{n,i}x, \quad i=1,2, \qquad (22)$$

Here M_1=0, as so as this mass is in coalface, (рис. 2).

Given below formulas are followed from the boundary and conjugation conditions (17) for two parts of the drill column

$$\begin{cases} A_{n,1}=0; \\ A_{n,1}\cos k_{n,1}l_1+B_{n,1}\sin k_{n,1}l_1=A_{n,2}\cos k_{n,2}l_1+B_{n,2}\sin k_{n,2}l_1; \\ E_1F_1k_{n,1}(-A_{n,1}\sin_{n,1}l_1+B_{n,1}\cos k_{n,1}l_1)=E_2F_2k_{n,2}(-A_{n,2}\sin_{n,2}l_1+B_{n,2}\cos k_{n,2}l_1); \\ -M_2\omega_n^2(A_{n,2}\cos k_{n,2}l+B_{n,2}\sin k_{n,2}l)+E_2F_2k_{n,2}(-A_{n,2}\sin_{n,2}l+B_{n,2}\cos k_{n,2}l)=0. \end{cases} \qquad (23)$$

After simplification the determinate of the homogeneous system (23) is put to zero, then the equation for determination of the eigenfunctions and the natural frequency of the drill column vibration are defined

$$\sin\lambda_n(\lambda_n\alpha\xi\cos\eta\lambda_n+\alpha^2\sin\eta\lambda_n)+\cos\lambda_n(\lambda_n\xi\sin\eta\lambda_n-\alpha\cos\eta\lambda_n)=0; \quad (24)$$

where $\lambda_n=\lambda_{n,1}=\dfrac{\lambda_{n,2}a_2l_1}{a_1l_2}$, $\eta=\dfrac{a_1l_2}{a_2l_1}$, $\alpha=\sqrt{\dfrac{E_2F_2m_2}{E_1F_1m_1}}$, $\xi=\dfrac{M_2}{m_1l_1}$.

Well-known special case of the homogeneous column [7] is fined from (24) under $\alpha=1,\ l_2=0,\ \ l_1=l$.

Longitudinal impact occurs under slackening of block-and-tackle system (which don't interact with column) for drilling rigs of rotary type or spindle drills and it is possible to put $\xi = 0$, therefore

$$\alpha \sin \lambda_n \sin \eta\lambda_n - \cos \lambda_n \cos \eta\lambda_n = 0 . \qquad (25)$$

Well-known frequency equation [1] is defined from (25) for special case of homogeneous column $(\alpha = 1,\ \eta = 0)$.

The coefficients $B_{n,1}$, $A_{n,1}$, $B_{n,2}$ are defined from the system (23). Taking into account that the eigenfunctions are defined accurate to constant, it is possible to put $B_{n,1} = 1$ and the following equation is defined from the system (23)

$$\begin{cases} A_{n,2} \cos k_{n,2} l_1 + B_{n,2} \sin k_{n,2} l_1 = \sin k_{n,1} l_1; \\ -A_{n,2} \alpha \sin k_{n,2_{n,2}} l_1 + B_{n,2} \alpha \cos k_{n,2} l_1 = \cos k_{n,1} l_1. \end{cases} \qquad (26)$$

The eigenfunctions are obtained from the solution of the system (26)

$$X_n(x) = \begin{cases} \sin k_{n,1} x; \quad 0 \le x \le l_1; \\ A_{n,2} \cos k_{n,2} x + B_{n,2} \sin k_{n,2} x, \quad l_1 < x \le l, \end{cases}$$

where:

$$A_{n,2} = \sin \lambda_n \cos \frac{\lambda_n a_1}{a_2} - \frac{1}{\alpha} \sin \frac{\lambda_n a_1}{a_2} \cos \lambda_n;$$

$$B_{n,2} = \frac{1}{\alpha} \cos \frac{\lambda_n a_1}{a_2} \cos \lambda_n + \sin \lambda_n \sin \frac{\lambda_n a_1}{a_2},$$

and proper numbers λ_n are defined from the system of equations (25).

Longitudinal displacements are produced as eigenfunctions expansion taking into account first initial condition (21)

$$u(x,t) = \sum_{n=1}^{\infty} C_n X_n(x) \sin \omega_n t. \qquad (27)$$

Then we should satisfy second initial conditions (21). The following equation is defined by Fourier method with weight according to scheme

$$C_n \Delta_n^2 \omega_n = -v_o \left(m_1 \int_0^{l_1} \sin k_{n,1} x dx + m_2 \int_{l_1}^{l} \left(A_{n,2} \cos k_{n,2} x + B_{n,2} \sin k_{n,2} x \right) dx \right), \qquad (28)$$

where

$$\Delta_n^2 = \frac{m_1 l_1}{2} \left(\cos^2 \lambda_n \left(1 - \frac{E_1 F_1}{E_2 F_2} \right) + \sin^2 \lambda_n \left(1 - \frac{m_2}{m_1} \right) \right) + \frac{m_2 l}{2} \left(A_{n,2} \cos \left(\frac{\lambda_n a_1 l}{a_2 l_1} \right) + B_{n,2} \sin \left(\frac{\lambda_n a_1 l}{a_2 l_1} \right) \right)^2 -$$

the squared norm of the eigenfunctions, which calculated by formula (19).

Value of series coefficients (27) are calculated from (28)

$$C_n = -\frac{V_o l_1^2}{\Delta_n^2 a_1 \lambda_n^2}\left(m_1(1-\cos\lambda_n) + \frac{m_2 a_2}{a_1}\left(\begin{array}{l} A_{n,2}\left(\sin\left(\frac{\lambda_n a_1 l}{a_2 l}\right) - \sin\left(\frac{\lambda_n a_1}{a_2}\right)\right) - \\ -B_{n,2}\left(\cos\left(\frac{\lambda_n a_1 l}{a_2 l_1}\right) - \cos\left(\frac{\lambda_n a_1}{a_2}\right)\right)\end{array}\right)\right).$$

So the stresses of the column are calculated from formula

$$\sigma(x,t) = \begin{cases} \sum_{n=1}^{\infty} E_1 C_n k_{n,1} \cos k_{n,1} x \sin\omega_n t, & 0 \le x \le l_1; \\ \sum_{n=1}^{\infty} E_2 C_n k_{n,2}(-A_{n,2}\sin k_{n,2}x + B_{n,2}\cos k_{n,2}x)\sin\omega_n t, & l_1 < x \le l. \end{cases} \quad (29)$$

Calculation stresses consist of two components: stresses from motion velocity variation $\sigma^{(1)}$ - (29) and stresses from sudden application of column weight - $\sigma^{(2)}$. Maximum value of second component of the stresses does not outnumber doubled value of static stresses [1] and calculation of them is not complicated procedure.

The variation of maximum nondimensional stresses $\bar{\sigma}_1 = \frac{\sigma_{1,\max} a_1}{E_1 v_o}$ from nondimensional time $\tau = \frac{a_1 t}{l_1}$ at $x = 0$ and drilling depth 200 m is presented on Fig. 3.

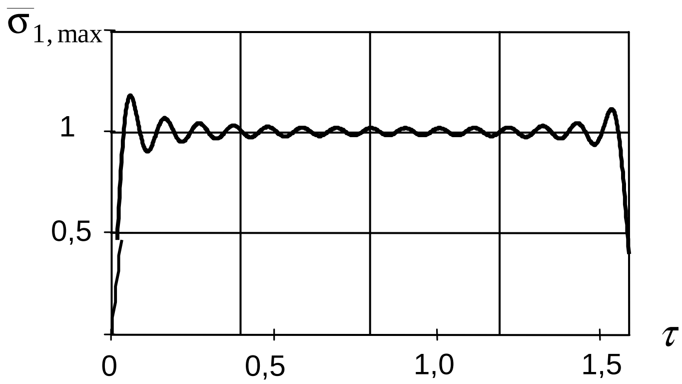

Fig. 3. The variation of maximum nondimensional stresses $\bar{\sigma}^{(1)}_{\max}$ at drilling depth 200 m

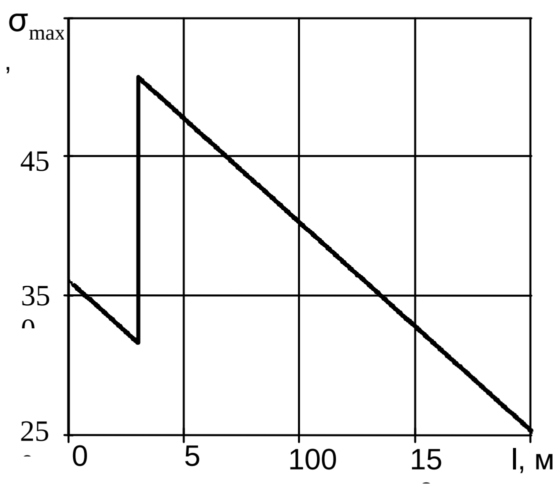

Fig. 4. The distribution of the maximum stresses

These stresses appear at the joint drill column and cutter. The parameters of the spindle drill column is used for the calculation.

It is evident from Fig. 3 that the behavior of the column stresses corresponds to impact processes nature.

The variation of total maximum dynamic stresses $\sigma_{\max} = \sigma_{1,\max} + \sigma_{2,\max}$ at velocity of column descending $v_0 = 2\ m/s$ and drilling depth 200 m is presented on Fig. 4.

Thus stressed-deformed state of double-step drill column of drilling rig under impact loads could be totally examined using proposed dependences and permissible operative conditions could be determined.

Literature

1. Тимошенко С.П. Колебания в инженерном деле. – М.: Наука, 1967 – 449 с.

2. Улитин Г.М., Петтик Ю.В. Собственные колебания балки ступенчато-переменного сечения//Збірник наукових праць. Серія: Галузеве машинобудування, будівництво - Вип. 16. – Полтава: ПолтНТУ, 2005. – 279-283.

3. Шевченко Ф.Л. Динаміка пружних стержньових систем. – Донецьк: ДонНТУ, 2000. – 293 с.

4. Улитин Г.М. К теории колебаний стержневых систем ступенчато-переменной жесткости//Автоматизація виробничих процесів у машинобудуванні та приладобудуванні. -Львів: „Львівська політехніка”. – 2006. - Вип. 40. – С. 250-254.

5. Арсенин В.Я. Методы математической физики и специальные функции. – М.: Наука. 1974. – 433 с.

6. Улитин Г.М., Петтик Ю.В. Математическая модель ударных процессов в двухступенчатых бурильных колоннах //Вибрация в технике и технологиях. -2007. -№3. - С. 26-29.

7. Улитин Г.М. Решение динамических задач на ударную загрузку в буровых установках роторного типа // Вибрация в технике и технологиях. -1998. -№2. - С. 78-80.