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Method for determining the trajectories of particle movement in the undermined rock mass

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Abstract. A method for determining the coordinates of particles displaced as a result of undermining a rock mass is considered, which makes it possible to pre-calculate displacements and deformations of a rock mass and the earth's surface. When compiling a model, a rock mass is considered as a discrete layered medium of a block structure. The mined seam is divided into elementary sections, which generate zones of displacement of rocks and the earth's surface. As a result of their addition, a common area of displacement is formed due to whole working area, and particles of rocks or the earth's surface move, forming displacement vectors. The direction and value of these vectors can be used to define changes in the position of the original line or surface and corresponding deformations caused by these changes. Mathematical modeling of the displacement process makes it possible to establish the patterns of spatial movement of points. Comparison of the calculated trajectories of points with the results obtained during mining operations relevealed their similarity. The proposed method can be used to predict the displacements of points in the undermined rock mass during the movement of the longwall face.

1. Introduction

The process of displacement of rocks is their movement and deformation as a result of imbalance caused by mining.

During the process of displacements, a change in the volume of rocks occurs, as a result of which zones of compression deformations are formed, corresponding to zones of increased rock pressure, as well as zones of loosening of rocks, corresponding to zones of decreased stress. Thus, the process of displacement of rocks is closely linked to the nature of the redistribution of stress in the rock mass and determines its stress-strain state, which must be taken into account choosing the parameters of development systems and most technological processes for performing of underground mining operations.

Movements and deformations can cause negative consequences for objects that are in the subsidence trough. Considering prediction of the magnitude of displacements and deformations, appropriate protection measures are selected.

The process of occurrence of displacements and deformations is linked to the movement of points in the subsidence trough. With movement of the point, displacements occurs: with vertical one – subsidence does, with horizontal movement – horizontal displacements does. As a result, the point traces out a spatial trajectory, the individual parts of which correspond to the position of the longwall face, which is in motion. Deformations occur when two adjacent points move asynchronously. Thus, knowing the spatial coordinates of two adjacent points at a certain position of the longwall face, it is possible to determine all types of displacements and deformations at the appropriate time.

Existing methods [1] make it possible to determine the displacements and deformations of the earth's surface only after the longwall stops, and only if displacement process ends.

Topic of determining the spatial trajectories of points in subsidence trough during the movement of the longwall face is almost not studied. There are several works devoted to it.

General appearance of the point`s trajectory during the movement of the longwall face is described in [2]. The point is located in the main section of the subsidence trough along the strike of the formation. Its subsidence and horizontal movements in a vertical plane parallel to the direction of movement of the longwall face are considered; the characteristic sections of the trajectory of the point's movement are highlighted depending on the position of the longwall face. The type of the trajectory of a point located in an arbitrary place of the subsidence trough is not defined in this work; trajectories in projections on the horizontal plane and perpendicular to the direction of movement are not considered.

The work [3] presents the results of observations of the movement of points in the undermined rock mass at different positions of the longwall face; graphs of subsidence and horizontal displacements in perpendicular and parallel directions relating to the movement of the longwall face are created; the trajectories of the points are not shown.

Works [4, 5] are linked to the study of the nature of horizontal displacements of points during the movement of the longwall face; the trajectories of point`s movement in a vertical plane parallel to the direction of movement of the longwall face are illustrated.

Thus, in the literature there are no results of creating the trajectories of points in projection onto a horizontal plane and a plane perpendicular to the direction of movement of the longwall face, as well as for points located at an arbitrary place in the subsidence trough. Furthermore, a method for predicting the displacements of points during the movement of a coal face is undeveloped, which makes it possible to construct their spatial trajectories.

During the process of longwall mining, magnitudes of point`s displacements may be greater than after the end of the displacement process. Therefore, for the timely implementation of measures for the protection of objects, it is necessary to predict the position of the points of the subsidence trough at different points in time, which determines the relevance of this study.

The aim of the work is to establish the spatial position of points in the contour of the subsidence trough to determine their trajectories during the movement of the longwall face.

2. Methods

To describe the process of moving particles, a mathematical model is used, based on the representation of a rock mass in the form of a discrete layered medium of a block structure.

Many researchers agree that rocks are initially divided into blocks by cracks, which are a complex hierarchical system from microcracks to large geological faults.

Cracks create surfaces of weakening, along which rocks are destroyed due to technogenic impact, forming blocks and, moving under the influence of gravity, filling the voids formed as a result of mining. Therefore, it is quite reasonable to assume that during the extraction of a coal seam, the overlying layers of rocks move in accordance with the regularities, inherent for particles of a granular medium.

The regularities of the movement of particles of bulk material is most fully studied modeling the processes of movement of blasted ore to the outlets under the collapsed overlying rocks in the works of V. V. Kulikov [6] and G. M. Malakhov.

According to V. V. Kulikov, the outflow of bulk materials stems from volumes that are close in shape to ellipsoids of revolution.

One of the properties of the outlet ellipsoid is the fact that particles located on its surface reach the outlet at the same time. The particles move towards the outlet in parabolic paths. When discharging, only a certain part of the bulk material moves which passes the stage of loosening and also has the shape of an ellipsoid of revolution, called the ellipsoid of loosening.

As soon as the bulk material is released, the ellipsoid of loosening increases, forming a paraboloid – the limiting boundary of the area under impact of the outlet beyond which the particles remain stationary when any amount of material is released.

In the volume of the ellipsoid of loosening, firstly horizontal surfaces acquire the shape of funnels, called deflection funnels. When the ellipsoid of loosening reaches the earth's surface, funnel or subsidence trough is formed on it.

The author suggests a mathematical model of the process of moving particles in an undermined rock mass when extracting a coal seam, based on the representation of the undermined rock mass in the form of the discrete medium [7].

The movement of particles occurs according to the following scheme (figure 1**)**.

When an element of a coal seam is removed, an elliptical zone of influence arises, in which, as a result of additional (secondary) loosening, ellipsoids of motion are formed. Particles located on the surface of such ellipsoids pass to the surface of the underlying ellipsoids, the volume of which is smaller in terms of the volume of the ellipsoid corresponding to the extracted element of the coal seam.

The generatrix of the ellipsoid is defined as the following equation:

$$y^2 = 2px\left(\frac{H_i - x}{H_i}\right), \tag{1}$$

where y – distance in the plane of the coal seam from the extracted element, m; x – height above coal seam, m; p – focal parameter of the ellipse, m; H_i – ellipse height, m.

The focal parameter p determines the shape of the ellipse. It is an integral indicator that takes into account the complex of physical and mechanical properties of rocks that have impact on their displacement and, therefore, is called the rock displacement index. The value of the rock displacement index p depends on the size of the rock blocks involved in the movement and is approximately equal to their threefold size.

For the conditions of the Donetsk coal basin, the rock displacement index can be equal to 40- 60 m, which corresponds to a threefold size of the average step of the collapse of the main roof of the extracted seam.

Particle movement is initiated by the extraction of a portion of the coal seam. The volume of the extracted element is filled with collapsed rocks. Consequently, an extraction ellipsoid is formed in the rock mass.

The volume of the extraction ellipsoid q (m^3), corresponding to the volume of the extracted element of the coal seam, is determined by the formula:

$$q = \frac{\pi p h^2}{3}, \tag{2}$$

where h – the height of the extraction ellipsoid, m.

The extracted element of a coal seam with a thickness of m (m) and an area S (m^2) is filled with rocks of the immediate roof, the volume of which is less than the volume of the extracted element due to their loosening.

Thus, the volume of the extraction ellipsoid will be equal to:

$$q = \frac{mS}{k_1}, \tag{3}$$

where k_1 – the coefficient of loosening of rocks; k_1 = 1.2 - 1.3.

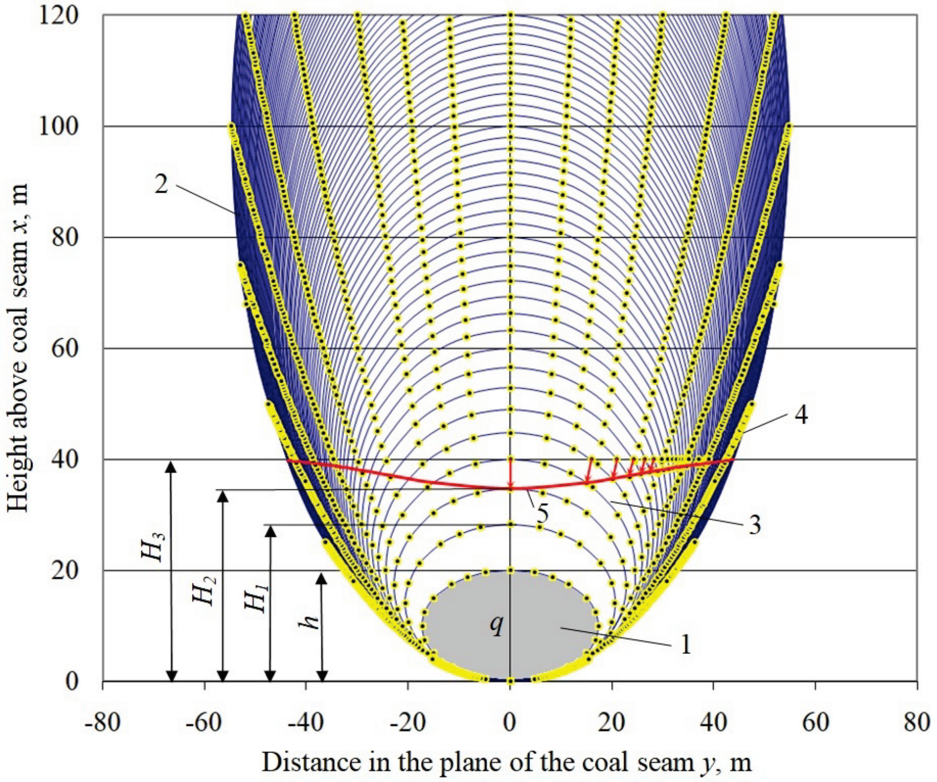

Figure 1. Trajectories of moving points during the movement of rocks and the earth's surface as a result of the extraction of an element of the coal seam: 1 – extraction ellipsoid, 2 – boundary of the influence zone of the extracted element, 3 – ellipsoid of motion, 4 – trajectories of moving points, 5 – subsidence trough.

Formulas (2), (3) enable us to find the height of the extraction ellipsoid using the initial data – the values of m, S, p, k_1:

$$h^2 = \frac{3mS}{\pi p k_1}. \tag{4}$$

Due to the extraction of an ellipsoid of volume q, an ellipsoid of secondary loosening of the rock mass arises in the overlying rocks, which is the influence zone of the extracted element of the coal seam.

The generatrix of this zone is described by the equation:

$$y^2 = 2px\left\{1 - x\left[h^2\left(\frac{k_2 - 1}{k_2}\right)\right]^{\frac{1}{2}}\right\}, \tag{5}$$

where k_2 – the coefficient of secondary loosening of the rock mass, determined from the expression:

$$k_2 = \frac{H_n{}^2}{H_n{}^2 - h^2}, \tag{6}$$

where H_n – the height of the loosening ellipsoid, m.

In conditions of coal seams mining, the coefficient of secondary loosening of the rock mass is close to 1. In this case, the generatrix of the influence zone of the extracted element of the coal seam will be described by the parabola equation:

$$y^2 = 2px. \tag{7}$$

There is ability to draw the generatrix of the ellipsoid of motion through any point in the zone of influence. The volume of the ellipsoid of motion Q_i with height H_i is equal to:

$$Q_i = \frac{\pi p H_i^2}{3}. \tag{8}$$

After extracting the element of the coal seam, the point with the x_i coordinate located on the surface of the ellipsoid of motion with the volume Q_i (m^3) will move to the underlying ellipsoid, the volume of which Q_{i-1} will be volume of the extraction ellipsoid q smaller:

$$Q_{i-1} = Q_i - q. \tag{9}$$

After moving to the surface of an ellipsoid with a volume of Q_{i-1} and a height of H_{i-1}, the point will have a coordinate x_{i-1}, determined from the ratio:

$$\frac{x_{i-1}}{H_{i-1}} = \frac{x_i}{H_i}. \tag{10}$$

Moving to the surface of the underlying ellipsoid, the point moves along a trajectory described by the parabola equation:

$$y_{i-1}^2 = \frac{x_{i-1} y_i^2}{x_i}, \tag{11}$$

where x_i, y_i – coordinates point located on the surface of the ellipsoid of motion with the volume Q_i, m; x_{i-1}, y_{i-1} – coordinates point located on the surface of the ellipsoid of motion with the volume Q_{i-1}, m.

The method for calculating the displacement of particles is as follows. The area of the mined coal seam is divided into sections which, after extraction, initiate the emergence of elliptical zones of influence of the extracted element of the coal seam. Points of the earth's surface and rock strata move if they take place in one or more zones of influence (figure 2).

Point A moves under the action of the extracted element of the coal seam 1 and occupies position A_1, and point C moves under the action of the extracted element of the coal seam 2 and occupies position C_2. Point B is influenced by the extracted elements of the coal seam 1 and 2. As a result, it is first moved to position B_1 and then to position B_2. Further movement of points occurs when mining other sections of the coal seam.

Common coordinate system *OXYZ* is chosen arbitrarily, for example, at the beginning of longwall mining. Each extracted element of the coal seam has a particular coordinate system in accordance with figure 1. The *X*-axis corresponds to the *Z*-axis in figure 2, and the *Y*-axis is directed from the center of the extracted coal seam element to a point. The coordinate of a point along the *Y* axis (figure 1) is projected onto the *X* and *Y* coordinate axes of the common coordinate system. Having done calculations, the coordinates of the point are converted to the general coordinate system *OXYZ*.

As a consequence of modeling, the initial coordinates of the particles are converted to output, and the initial position of the line or any surface acquire a transformed form. Modeling, you can specify any sequence of extraction of coal seam elements.

This scheme of particle movement in the specified mathematical model is used in a rock mass up to a certain height above the coal seam.

Comparing the simulation results with actual data, it was found that horizontal displacements and deformations on the earth's surface are several times greater than in the rock mass. That is, in the near-surface zone, including sediments, particles move along more gentle trajectories.

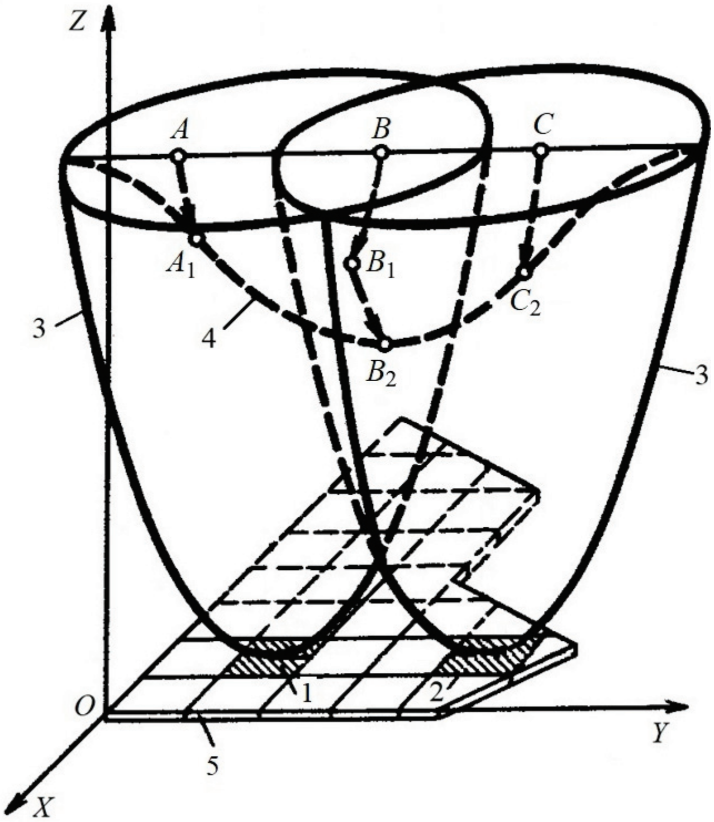

Figure 2. The scheme of displacement of points during the movement of rocks and the earth's surface as a result of the extraction of a coal seam: 1, 2 – extracted elements of the coal seam; 3 – boundary of the influence zone of the extracted element; 4 – subsidence trough; 5 – coal seam; *A*, *B*, *C* – initial position of points; A_1, B_1, B_2, C_2 – position of the points after moving.

This feature is also noted in works [8-10]. There is assumption that the near-surface zone is a free-lying beam about 50 m thick, not loaded by the overlying layers of rocks. In this zone, size of horizontal displacements and deformations gradually increase and on the surface exceed the values determined in the rock mass by several times [8]. The thickness of the layers into which the undermined rock mass is stratified is 5 - 10 m [9, 10]. Considering that the horizontal displacements of points, according to the common hypothesis, are proportional to the thickness of the bending layers, the horizontal displacements of points on the earth's surface will be 5-10 times greater than in the rock mass. In the suggested mathematical model, the increase in horizontal displacements and deformations on the earth's surface is taken into account by introducing sediment coefficient. Its value is 5-7. It depends on the specific mining and geological conditions. The calculated values of subsidence, inclinations, curvatures for the earth's surface are taken the same as for the rock mass.

Described mathematical model has some similarities with the well-known method of pre-calculation of displacements and deformations of the earth's surface called "influence function" [11, 12]. They are similar in the fact that the extracted layer is divided into sections that affect the earth's surface, causing displacement and deformation in a certain area. The differences are as follows. When using the influence function method, the graphs of displacements and deformations during longwall mining are obtained by summing the elementary troughs arising from the action of individual sections. Strain values are obtaining as a result of mathematical transformations of the original equation of subsidence. The suggested mathematical model does not use the original equations for forming displacement and deformation graphs. They are based on coordinates of the displaced particles.

A characteristic feature and advantage of the suggested mathematical model is the ability to obtain the spatial coordinates of any point in the area of displacement of rocks after movement under the influence of the goaf, which makes it possible to construct a vector of its movement. Knowing the coordinates of the points that have moved, as a result of the displacement of the rock mass, it is possible to determine all types of displacements and deformations.

The initial data for modeling are:

- mining parameters (the size of the goaf along the strike and dip);
- mining and geological factors (thickness, dip angle, bed depth, rock displacement index – an integral parameter that takes into account the properties of the rock mass, affecting its movement);
- geometric parameters of modeling (the position of the points of the profile line in space, determined by the coordinates of the starting point, the turning angle and the distance between the points, the dimensions of the extracted elements of the coal seam initiating the displacement process, the angle of the direction of the area of influence);
- additional parameters (direction of movement of the longwall face, influence of sediments, distances between horizontal and vertical sections for calculating vertical deformations or volumes of the displacement trough).

The described mathematical model can be used not only for pre-calculation of displacements and deformations during longwall mining, but also for studying the displacement process, clarifying existing and establishing new patterns of this process. For example, in [13], using this model, a study was carried out to determine the location of the inflection points of the subsidence curve.

3. Results and discussion

In order to study the patterns of movement of points during the movement of the longwall face, mathematical modeling was done, using the described method. As a result of modeling, the coordinates of the trajectory of movement of one of the points in the subsidence trough were obtained. Modeling was carried out in the following conditions: coal seam thickness 2 m; dip angle 0°; development depth 400 m; the size of the longwall along the strike is 400 m, in the perpendicular direction - 200 m. The point was located for a distance of 200 m from the beginning of longwall mining and for a distance of 50 m from the axis of the longwall (figure 3).

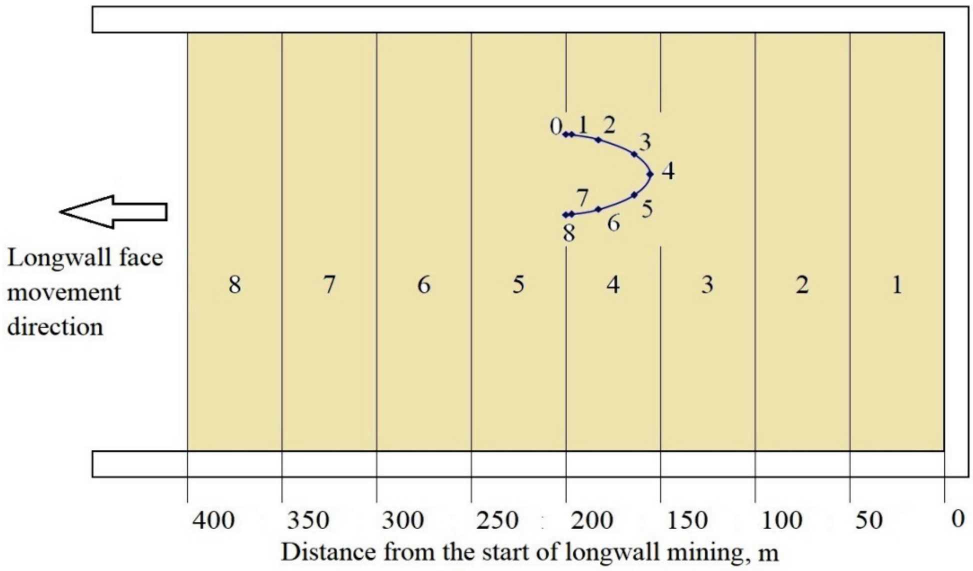

Figure 3. Schematic trajectory of a point on the plan of the longwall being worked out: 0 – 8 – the position of the point at the corresponding moment of the longwall mining.

The nature of the movement of the point during movement of longwall face in the projection onto the horizontal plane has been established.

As can be seen from figure 3, when the longwall face moves, the point traces out a parabolic trajectory, moving horizontally parallel to the movement of the longwall face and towards the middle of the longwall. There is also a change in the magnitude and direction of horizontal displacements. Initially, the point moves in the opposite direction to the movement of the longwall. The greatest horizontal movement in this direction occurs in section 2-3. Moving to the middle of the longwall, in position 4, when the longwall face is directly below the point, it changes its direction of movement and begins to move in the same direction with the longwall face. In position 8, when the mined coal seam no longer has impact on point, the movement of the point ends.

Described character of the point movement is explained by the number and location of the elliptical zones of influence in which this point takes place. As more zones will affect the point, greater horizontal displacements will be, displacing it in this direction.

For a more detailed study of the trajectory of the point movement during the movement of the longwall face, graphs of the trajectory in 3 projections, as well as the spatial trajectory were built (figures 4-7).

When constructing graphs, three directions of point displacement were taken into account in various combinations:

- horizontal displacements of the point in parallel and perpendicular to the direction of movement of the longwall face – projection onto the horizontal plane (figure 4);
- horizontal displacements perpendicular to the direction of movement of the longwall face and vertical displacements – projection onto the frontal plane (figure 5);
- horizontal displacements parallel to the direction of movement of the longwall face and vertical displacements – projection onto a vertical plane parallel to the direction of movement of the face (figure 6).

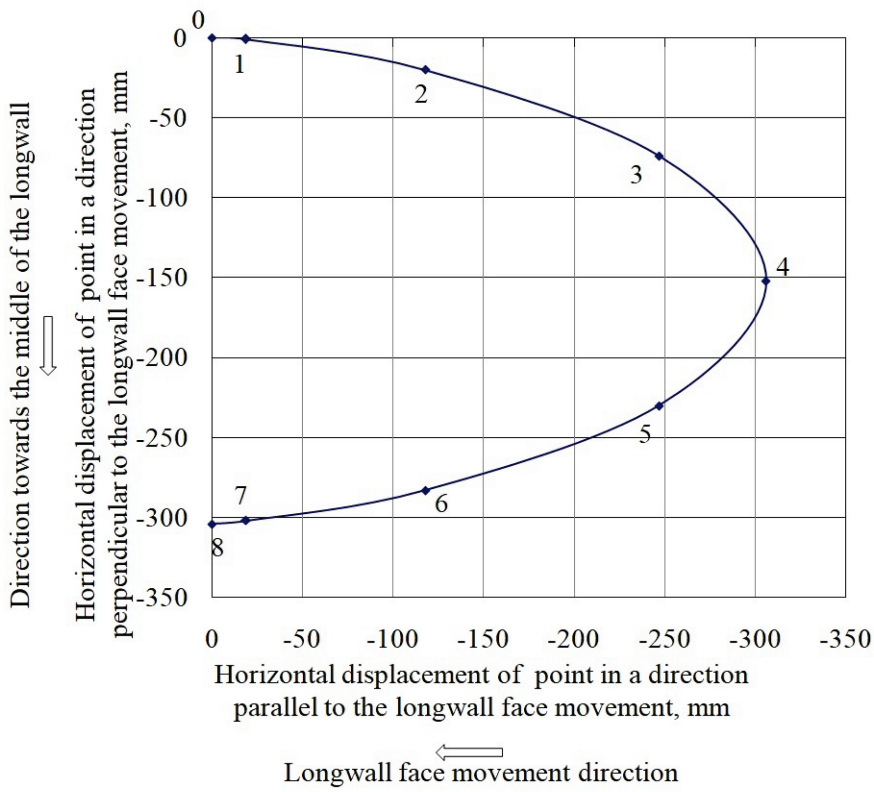

Figure 4. Trajectory of moving a point in projection onto a horizontal plane.

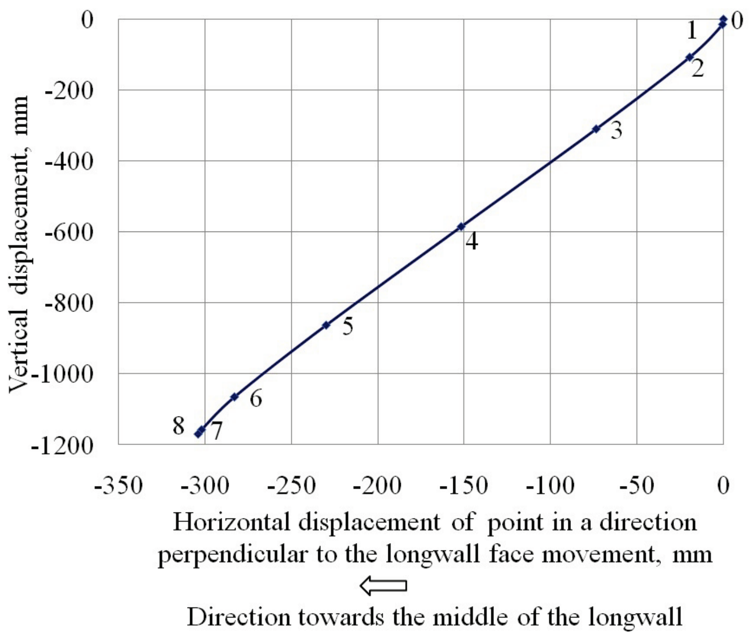

Figure 5. Trajectory of moving a point in projection onto the frontal plane.

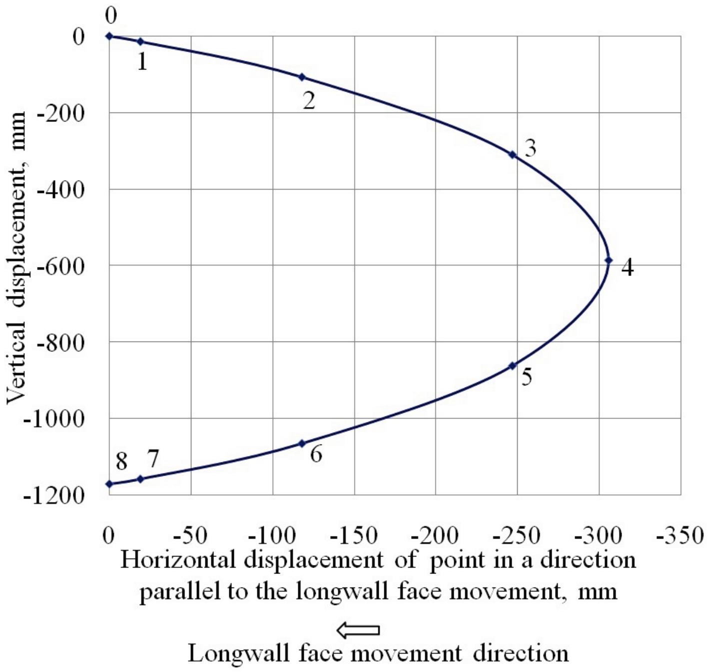

Figure 6. The trajectory of moving a point in projection onto a vertical plane parallel to the direction of of the longwall face movement.

Analyzing the trajectories of the points shown in figures 4 and 6, first of all, their parabolic appearance attracts attention.

Moving in a horizontal plane and in a vertical plane parallel to the direction of movement of the longwall face, the point traces out the trajectory of the parabola. The vertices of the parabolas are at position 4, which corresponds to the position of the face of the longwall directly below the point.

There is a change in the horizontal direction of moving the point to the opposite. In the perpendicular direction, the point continues to move towards middle of the longwall.

Trajectory positions 0-8, 1-7, 2-6, 3-5 are located symmetrically relating to the parabola axis and correspond to the positions of the longwall face, which are at the same distance from the moving point. The largest displacements in the horizontal direction were obtained when the longwall face moved from positions 2, 5 to positions 3, 6, at distances in the plan of 50 - 100 m from the point.

On the trajectory sections corresponding to positions 3, 4 and 4, 5 at a distance from the point 0- 50 m, there is a noticeable increase in the values of vertical displacements, as well as horizontal displacements towards the center of the longwall. In these areas, the trajectory curve becomes steeper and changes its direction. After passing the face of the longwall under the point (position 4), it moves along the trajectory corresponding to the second, symmetrical branch of the parabola. The movements stop when the point of maximum subsidence is reached, that is, when the longwall face moves a distance exceeding the radius of influence of the extracted elements of the coal seam. The graphs presented allow you to determine the nature of the change in the horizontal and vertical displacements of the point, depending on the position of the longwall face.

The trajectory of the displacement of the point obtained in the simulation during the movement of the longwall face is quite accurately described by the parabola equation (the accuracy of the approximation is $R^2 = 0.982$).

Result coordinates of the point obtained during the simulation made it possible to construct a spatial trajectory, which clearly demonstrates the nature and features of its movement during the movement of the longwall face (figure 7).

The spatial image of the trajectory makes it possible to match and link the movement of a point in all directions.

In projection onto the vertical (frontal) plane, the trajectory of the point movement has the form of a straight line segment, located at an angle of inclination to the horizontal plane. It indicates that the spatial trajectory of the point's movement is a flat curve.

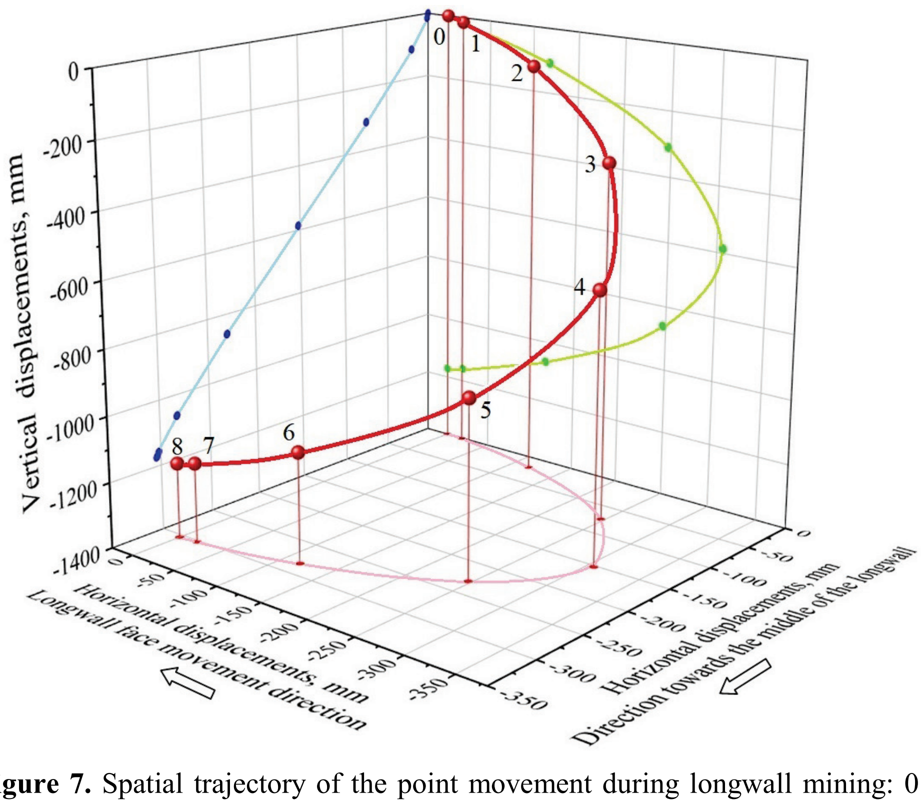

Figure 7. Spatial trajectory of the point movement during longwall mining: 0 – 8 – point positions corresponding to the movement of the longwall face through 50 m.

The angle of inclination of the plane of the trajectory depends on the location of the point relating to the longitudinal axis of the longwall. It will increase as the point approaches the center of the longwall, reaching a maximum value of 90°, which is explained by the equal degree of influence of the worked out sections of the longwall around the point. In this case, the projection of the trajectory of the point on the horizontal and frontal planes will have the form of straight line segments, and in the plane of the vertical projection parallel to the direction of movement of the lonwall face, the trajectory will be displayed in the form of a curve. This case is often encountered in the practice of observing the displacement of the earth's surface, when the observation station is laid along the axis of the subsidence trough.

Describing the trajectory of moving a point, depending on the position of the face of the longwall in [2], several characteristic sections are distinguished (figure 8, a): I - the face is approaching the point; II - the face passes under the point; III - the face moves away from the point; IV - subsidence of the point as a result of compaction of rocks after the end of the influence of the extracted coal seam. Sometimes vertical displacements of a point corresponding to section II may be absent.

The trajectory of the point, built according to the results of the performed mathematical modeling, is similar in appearance to the described scheme, only section IV is absent. This is due to the fact that the stage of damping of the displacement process was not identified in the simulation. Whilst, there is data of field observations that correspond to the results of mathematical modeling (figure 8, b). The figure shows the trajectories of movement of benchmarks located in the central part of the longwall during the movement of the longwall face [5]. When comparing the actual trajectories with the trajectory obtained as a result of modeling (figure 6), it can be noted that they are quite similar.

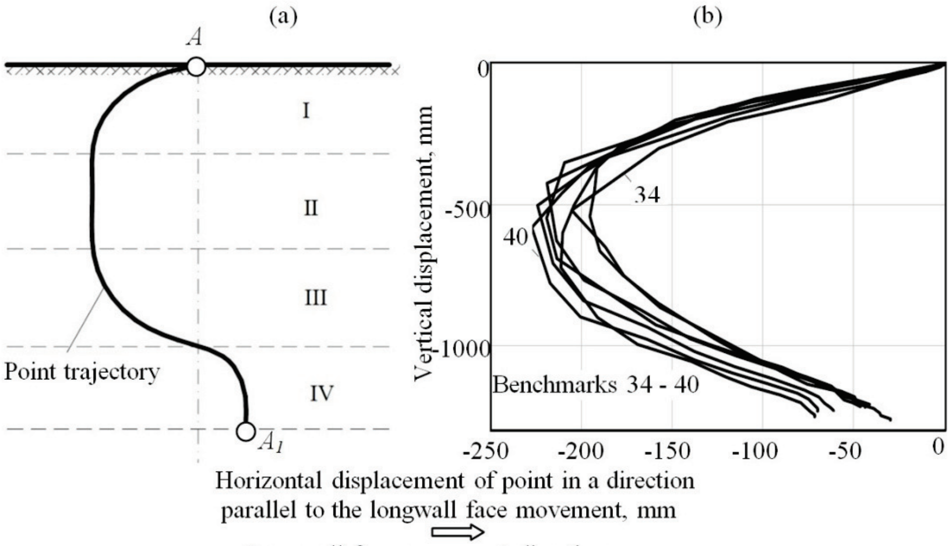

Figure 8. Diagram of typical sections of the trajectory of the point movement when moving the longwall face (a), [2] and the results of observations of the movement of benchmarks (b), [5].

The shape of the trajectory of movement of the point strongly depends on the speed of movement of the longwall face.

In works [14, 15] it is suggested that for all specific mining and geological conditions there should be such rate of movement of the longwall face, at which the displacements and deformations of the dynamic subsidence trough become equal to the displacements and deformations of the finally formed subsidence trough.

In order to establish the possibility of using the developed mathematical model for constructing the trajectory of moving a point in relation to real conditions, the simulation results were compared with the data obtained at the observation station of the Stepnaya mine [4].

Observation station No. 13 consisted of 52 benchmarks located above the center 604 of the longwall parallel to the direction of movement of the longwall face.

Modeling was carried out for benchmark No. 25, which was located at a distance of 30 m from the beginning of longwall mining in the direction of its movement.

The simulation determined the subsidence and horizontal displacement of the point in the direction parallel to the movement of the longwall face. The interval for determining the displacements was 10 m of longwall face movement.

Point trajectories constructed from simulation results and actual data are shown in figure 9.

In the course of modeling, the actual mining-geological and mining-technical conditions of development were taken: the thickness of the coal seam is 1 m; dip angle 5°; development depth 120 m; face length 180 m; the direction of longwall mining - along the uprising of the seam.

The following parameters of the mathematical model are accepted. The rock displacement index is 40 m, the sediment coefficient is 6, the area of the extracted element of the coal seam is 4 m^2, the direction of the elliptical zones of influence of the extracted elements is perpendicular to the bedding of rocks.

The trajectories shown in the figure 9 are close enough and have a parabolic shape. The accuracy of approximation by the parabola equation for the trajectory constructed from the simulation results is $R^2 = 0.997$, according to the actual data – $R^2 = 0.993$.

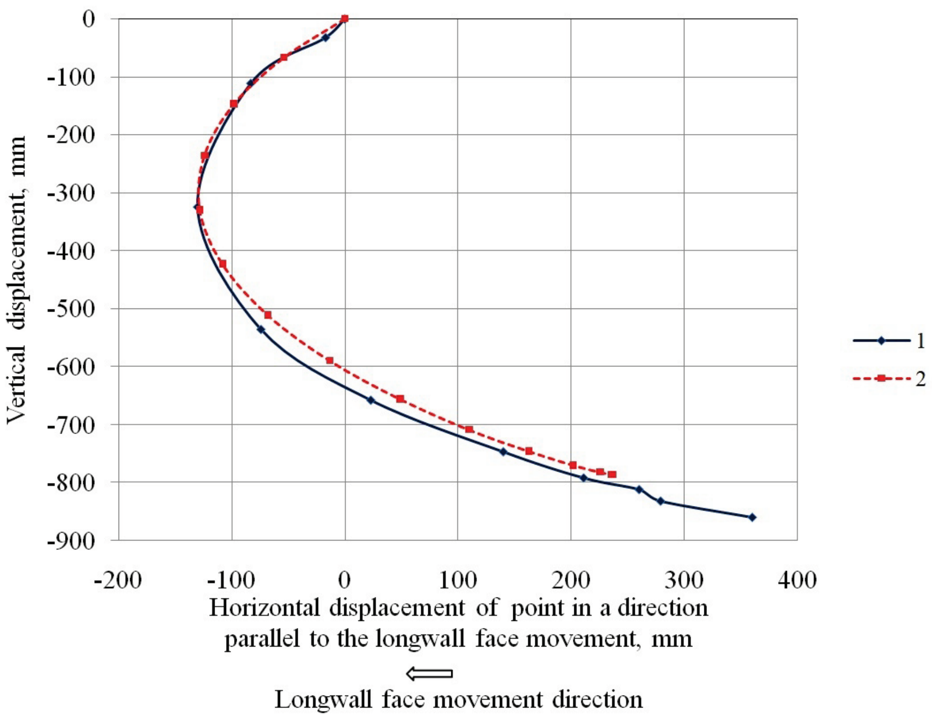

Figure 9. Comparison of the actual displacements of the point when moving the longwall face with the simulation data: 1 - actual data; 2 - simulation data.

A slight lag in vertical and horizontal displacements according to the modeling results can be explained by insufficient consideration of the influence of self-compacting of rocks.

4. Conclusions

A mathematical model of the movement of particles in an undermined rock mass is suggested, based on its representation in the shape of a discrete medium.

This model makes it possible to obtain the spatial coordinates of particles displaced due to displacement of rocks. Moreover, it makes it possible to determine all types of displacements and deformations of the rock mass and the earth's surface.

As a result of the application of this model, for the first time, the possibility of constructing the spatial trajectories of particles moving during the movement of the longwall face was obtained. It was found that the trajectories of the particles are described by the parabola equation quite accurately. The main regularities of the formation of these trajectories are determined depending on the location of the point and longwall face.

Comparison of simulation results with actual data is carried out and their convergence is noted. The resulting deviations could be caused by insufficient attention payed on modeling the impact of the stage of attenuation of the displacement process, accompanied by self-compaction of the rocks of the undermined rock mass.

To be able to take its influence in terms of modeling into account, it is further research that is essential.

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