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half-line with the Robin boundary condition. On the other hand, in the case of general initial data, no results are known.

Our paper can be viewed as an important step in solving the above-mentioned nonlinear problem with general initial data. We treat this system by the unified approach to IBV problems for linear and integrable nonlinear equations, also known as the Fokas unified transform method. Following the ideas of this method, we obtain the integral representation of the solution of the initial value problem, which can be efficiently used in the analysis of the nonlinear problem.

On the mappings in the class $W^{1,1}_{\rm loc}$ on Riemann surfaces

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In terms of dilatations K_f, it is proved a series of criteria for homeomorphic extension to the boundary of mappings f with finite distortion between regular domains on the Riemann surfaces, see for definitions [1] and [2]. For example:

Theorem 1. *Let $\mathbb{S}$ and $\mathbb{S}^*$ be Riemann surfaces, D and D^* be domains in $\overline{\mathbb{S}}$ and $\overline{\mathbb{S}^*}$, correspondingly, $\partial D \subset \mathbb{S}$ and $\partial D^* \subset \mathbb{S}^*$, D be locally connected on the boundary, and ∂D^* be weakly flat, and let $f : D \to D^*$ be a homeomorphism of finite distortion with $K_f \in L^1_{\rm loc}$. Suppose that, for every point $p_0 \in \partial D$ with the local coordinate z_0 in some chart U of the surface $\mathbb{S}$,*

$$\int_0^{\delta} \frac{dr}{\|K_f\|(z_0, r)} = \infty$$

for all small enough $\delta > 0$ where

$$\|K_f\|(z_0, r) \ = \int_{|z-z_0|=r} K_f(z)\,|dz|.$$

Then the mapping f is extended to the homeomorphism of $\overline{D}$ onto $\overline{D^*}$.

Here we assume that K_f is extended by zero outside of the domain D.

Corollary 1. *In particular, the conclusion of Theorem holds if for every point $p_0 \in \partial D$ with the local coordinate z_0 in a chart U of the surface $\mathbb{S}$,*

$$K_f(z) = O\left(\log \frac{1}{|z - z_0|}\right) \qquad \text{as} \quad z \to z_0$$

or, more generally,

$$k_{z_0}(\varepsilon) = O\left(\log\frac{1}{\varepsilon}\right) \qquad \textit{as} \quad \varepsilon \to 0$$

where $k_{z_0}(\varepsilon)$ is the mean value of the function K_f over the circle $|z - z_0| = \varepsilon$.

[1] S.V. Volkov, V.I. Ryazanov, *On the boundary behavior of mappings in the Sobolev class on Riemann surfaces*, Proceedings of Inst. Appl. Math. Mech. Nat. Acad. Sci. Ukr. **29** (2015), 34-53 (in Russian).

[2] S. Volkov, V. Ryazanov, *On the boundary behavior of mappings in the Sobolev class on Riemann surfaces*, ArXiv: 1604.00280v3 [math.CV] 27 Apr 2016, 27 pp.

On the Neumann problem for $A-$harmonic functions

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PDE's in the divergence form below take a significant part in many problems of mathematical physics in anisotropic and inhomogeneous media. Given a domain D in $\mathbb{C}$ and a real $(2 \times 2)-$matrix function A in D, a continuous function $u : D \to \mathbb{R}$ is called *$A-$harmonic function* if u is a generalized solution of the equation $\operatorname{div}(A\nabla u) = 0$. The matrix function $A(z)$ is called of *class $\mathcal{B}$* if $A(z)$ is measurable in z, symmetric, $\det A(z) \equiv 1$ and the given equation is uniformly elliptic. The following result was recently proved in [1].

Theorem 1. *Let D be a smooth Jordan domain in $\mathbb{C}$ whose unit interior normal $n = n(\zeta)$ at $\zeta \in \partial D$ is of bounded variation, $A(z)$, $z \in D$, be a matrix function of class $\mathcal{B} \cap C^\alpha$, $\alpha \in (0,1)$, and let a function $\varphi : \partial D \to \mathbb{R}$ be measurable with respect to logarithmic capacity. Then there exist A-harmonic functions $u : D \to \mathbb{R}$ of the class $C^{1+\alpha}$ such that, for a.e. point $\zeta \in \partial D$ with respect to the logarithmic capacity, there exist:*

1) the finite normal limit

$$u(\zeta) := \lim_{z\to\zeta} u(z) \ ,$$

2) the normal derivative

$$\frac{\partial u}{\partial n}(\zeta) := \lim_{t\to 0}\frac{u(\zeta + t\cdot n) - u(\zeta)}{t} = \varphi(\zeta),$$

3) the nontangent limit

$$\lim_{z\to\zeta}\frac{\partial u}{\partial n}(z) = \frac{\partial u}{\partial n}(\zeta).$$