On the solvability of third boundary value problem for improperly elliptic equation

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The solvability of inhomogeneous third boundary value problem (1) in a unit disk for second order scalar improperly elliptic differential equation (2) with complex coefficients and homogeneous symbol is studied. Such problem has a unique solution in the Sobolev space provided the boundary data belong to the space of functions with exponential decreasing of the Fourier coefficients.

We consider the problem

$$(u'_{\nu_*} - gu)|_{\partial K} = \beta, \tag{1}$$

$$Lu \equiv \left(\sin\varphi_1 \frac{\partial}{\partial x_1} + \cos\varphi_1 \frac{\partial}{\partial x_2}\right)\left(\sin\varphi_2 \frac{\partial}{\partial x_1} + \cos\varphi_2 \frac{\partial}{\partial x_2}\right)u = 0 \tag{2}$$

with complex angles φ_1 and φ_2, where K is a unit disk, $\frac{\partial}{\partial\nu_*}$ is a conormal derivative, $\beta \in H^m_\rho(\partial K)$, $g \in \mathbb{C}\backslash\{0\}$. Besides, we define $H^m_\rho(\partial K)$ as a Sobolev space with weight $\rho = \rho(n)$ to be consisted of the functions

$$\alpha(\tau) = \sum_{n=1}^{\infty}\left(\alpha_n^C \cos n\tau + \alpha_n^S \sin n\tau\right)$$

from $L_2(\partial K)$ such that

$$\sum_{n=1}^{\infty}\left(|\alpha_n^C|^2 + |\alpha_n^S|^2\right)\rho^2(n)\left(1+n^2\right)^m < \infty.$$

Additionally, we put

$$\rho = \rho(n) = e^{n(|\mathrm{Im}(\varphi_1+\varphi_2)| - |\mathrm{Im}(\varphi_2-\varphi_1)|)},$$

where $|\mathrm{Im}(\varphi_1+\varphi_2)| - |\mathrm{Im}(\varphi_2-\varphi_1)| > 0$ for improperly elliptic equation (2).

Theorem. *Let the angle $\varphi_0 = \varphi_2 - \varphi_1$ between characteristics be complex and $\beta \in H^m_\rho(\partial K)$. Then there exists the unique solution $u(x)$ of (1), (2), which belongs to $H^{m+3/2}(K)$.*