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PNEUMATIC CONVEYING FLOWS MODELLING BY THE SIMILARITY THEORY METHODS

At modeling complex technological processes methods of the theory of similarity can be used. Advantages of these methods are shown by the example of the equation of a homogeneous pneumotransport flow movement.

While complex technology processes modeling, the question of the variables considerable number of the process under consideration becomes of significant importance. With a big number of variables it is sometimes impossible to identify hidden relationships between them, build the system of equations and solve.

It is known that the influence of certain factors presented by different values becomes apparent not separately, but collectively. As a matter of fact not individual the values, but their collections determined for each process should be under consideration. In the theory of similarity the method of the collections building up has been developed. It allows identifying relationships between separate value groups and combining them into dimensionless groups of strictly defined type on the basis of the problem set up analysis. Being stable combinations of values sufficient for the processes under consideration, the groups get variables values of special kind, typical for these processes.

Moving from ordinary physical quantities to values of complex type offers distinct advantages. Variables number decreasing is achieved. While researching the problem in the values, reflecting certain factors influence as whole, inner relationships that characterize the process appear more distinct. New variables have one more important feature: not a single case, but many of them, combined by a property similarity, are under consideration. New variables are generalized, in fact. The use of them generalizes the whole analysis and enables considerable sharpening of the research results.

The shown advantages of complex variables create favorable conditions for solving tasks dealing with pneumatic conveying complex processes. First, in cases when it is possible to describe the process by means of the differential equations system, but the system remains non-closed. The substitution of the simple variables in the system for complex ones allows reducing the number of parameters and simplifies the problem solution. Second, if the process mathematical modeling is impossible, having the essential variable values list and with the help of the dimensional analysisprocess it is possible to create the so called determinantal equations of the process with the complex variables. And finally, the theory of similarity allows generalizing and expanding of empirical dependences use for a wider group of process and conditions, similar to those of the experiment.

As an example let us set up an equation of pressure losses while pneumatic conveying of fine disperse material through a horizontal pipeline under the constant temperature conditions (isothermal process).

According to the research, pressure losses occurring while dust even motion depend upon the following factors: the material mass flow rate M_M^*, the air flow velocity V_A the material particles density ρ_M, the air density ρ_A, the air kinematic viscosity v, solids effective diameter d_E,the pipeline diameter D, the pipeline segment length L, the material particles terminal velocity V_{Term}, gravitational acceleration g. In this case the variables implicit dependence will be expressed with the following equation:

$$\Delta P = f\left(M_M^*, V_A, \rho_M, \rho_A, v, d_E, D, L, V_{Term}, g\right). \quad (1)$$

According to the Buckingham π theorem any equation, combining n physical quantities (for example, velocity, viscosity, density and so on) among which m quantities are independent dimensionalities (for example, mass, length, time), can be transformed into the equation, combining $(n-m)$ dimensionless complexes (criteria) and simplexes derived from the quantities.

The π theorem allows finding relations not between separate physical quantities but their dimensionless ratios (π), set up by certain laws. At that the number of variables decreases by the number of used units of measurement.

It is also proved that expressing functional relationships between the quantities comprised in the equation (1) can be carried out in the form of the products of the exponents of the variables comprised in [1, 2]:

$$\Delta P = M_M^{*a_1} \cdot V_A^{a_2} \cdot \rho_M^{a_3} \cdot \rho_A^{a_4} \cdot v^{a_5} \cdot d_E^{a_6} \cdot D^{a_7} \cdot L^{a_8} \cdot V_{Term}^{a_9} \cdot g^{a_{10}}. \quad (2)$$

Function (2) can be transformed into the dimensionless variables dependency by the dimensional method:

$$\pi = C \cdot \Delta P^a \cdot M_M^{*b} \cdot V_A^c \cdot \rho_M^d \cdot \rho_A^e \cdot v^f \cdot d_E^i \cdot D^k \cdot L^m \cdot V_{Term}^n \cdot g^q, \quad (3)$$

where C – dimensionless constant;

$a, b, c, d, f, i, k, m, n, q$ – exponents.

The dimensionality of all the quantities comprised in (3) can be expressed by means of three main variables M, L, and T (mass, length, time). The temperature is not taken into account as the process is isothermal.

Let us consider the variables dimensionalities

$$[\Delta P] = \left[\frac{\text{kg}}{\text{sec}^2\,\text{m}}\right]; \quad [\nu] = \left[\frac{\text{m}^2}{\text{sec}}\right]; \quad [g] = \left[\frac{\text{m}^2}{\text{sec}}\right]; \quad [\rho_M] = \left[\frac{\text{kg}}{\text{m}^3}\right];$$

$$[M_M^*] = \left[\frac{\text{kg}}{\text{sec}}\right]; \quad [d_E] = [\text{m}]; \quad [V_{Term}] = [\text{m}]; \quad [L] = [\text{m}];$$

$$[V_A] = \left[\frac{\text{m}}{\text{sec}}\right]; \quad [D] = [\text{m}]; \quad [\rho_A] = \left[\frac{\text{kg}}{\text{m}^3}\right].$$

The exponents of the dimensionalities of the variables comprised in (3) are combined into the matrix of dimensionalities:

	$[\Delta P]$	$[M_M^*]$	$[V_A]$	$[\rho_M]$	$[\rho_A]$	$[\nu]$	$[d_E]$	$[D]$	$[L]$	$[V_{Term}]$	$[g]$
$[M]$	1	1	–	1	1	–	–	–	–	–	–
$[L]$	–1	–	1	–3	–3	2	1	1	1	1	1
$[t]$	–2	–1	–1	–	–	–1	–	–	–	–1	–2
r	a	b	c	d	e	f	i	k	m	n	q

Note: r *– the exponent.*

The exponent r can be designed by the matrix elements, having set up the equations linear system:

$$\left.\begin{array}{ll}\text{for }[M] & a+b+d+e=0 \\ \text{for }[\mathrm{L}] & -a+c-3d-3e+2f+i+k+m+n+q=0 \\ \text{for }[t] & -2a-b-c-f-n-2q=0\end{array}\right\} \quad (4)$$

According to the π theorem the number of expected dimensionless complexes is the difference between the columns and rows number of the dimensionalities matrix, i. e.11 – 3 = 8.

That is why for every type of dimensionless complex determining in this case any eight exponents values can be chosen. The values are chosen on reasonable basis.

For example, for determining the first dimensionless complex let a = 1 as $[\Delta P]$ is the unknown quantity. Let also $b=d=f=i=m=n=q=0$. Thereafter the system (4) solution gives $c=-2, e=-1$ и $k=0$.

The rest of 7 complexes are analogous designed.

As the result we obtain the solution matrix:

	$[\Delta P]$	$[M_M^*]$	$[V_A]$	$[\rho_M]$	$[\rho_A]$	$[\nu]$	$[d_E]$	$[D]$	$[L]$	$[V_{Term}]$	$[g]$
π_1	1	0	–2	0	–1	0	0	0	0	0	0
π_2	0	1	–1	–1	–1	0	0	–2	0	0	0
π_3	0	0	1	0	0	–1	0	1	0	0	0
π_4	0	0	0	1	–1	0	0	0	0	0	0
π_5	0	0	0	0	0	0	1	–1	0	0	0
π_6	0	–1/2	1/2	1/2	0	0	0	0	1	0	0
π_7	0	–1	0	1	0	0	0	2	0	1	0
π_8	0	0	–2	0	0	0	0	1	0	0	1

We obtain eight dimensionless complexes for each solution matrix row, including:

$\pi_1=\dfrac{\Delta P}{\rho_A V_A^2}$ – Euler criterion;

$\pi_2=\dfrac{M_M^*}{\rho_A V_A D^2}=0,785\mu$ – flow concentration;

$\pi_3=\dfrac{V_A D^2}{\nu}$ – Reynolds criterion;

$\pi_4=\dfrac{\rho_M}{\rho_A}=\rho$ – material particles relative density;

$\pi_5=\dfrac{d_E}{D}=d$ – material particles relative size;

$\pi_6=L\cdot\sqrt{\dfrac{V_M\rho_M}{M_M^*}}=\dfrac{L}{D}=l$ – pipeline relative density;;

$\pi_7=\dfrac{V_{Term}D^2\rho_A}{M_M^*}=\dfrac{V_{Term}}{V_M}=v$ – relative terminal velocity;

$\pi_8=\dfrac{gD}{V_A^2}$ – Froude number;

Thus the dependency (1) in the criteria form can be expressed the following way

$$E_u = f\left(\mu, \mathrm{Re}, \rho, d, l, v, Fr\right). \tag{5}$$

According to the group theory the dependency (5) can be transformed the following way:

$$E_u = C \cdot \mathrm{Re}^{b_1} \cdot Fr^{b_2} \cdot \mu^{b_3} \cdot \rho^{b_4} \cdot d^{b_5} \cdot l^{b_6} \cdot v^{b_7}. \tag{6}$$

The constant C and exponent b_i numerical values cannot be defined by the similarity theory methods, but require experimental determination.

While solving certain tasks, such as PCS engineering, when the parameters like ρ, d, and l, comprised in the equation (6) can be determined by other methods, the number of independent variables can be reduced up to four. In this case pneumatic conveying flows research is simplified.

The equation (6) can be used for the analysis of the processes of pneumatic conveying in the wide range of essential parameters change.

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